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THE EFFECT OF A SHORT WAVELENGTH MODE ON THE NONLINEAR EVOLUTION OF A LONG WAVELENGTH PERTURBATION DRIVEN BY A STRONG BLAST WAVE[a]

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We present a computational study of the formation of jets at strongly driven hydrodynamically unstable interfaces, and the interaction of these jets with one another and with developing spikes and bubbles. This provides a nonlinear spike-spike and spike-bubble interaction mechanism that can have a significant impact on the large-scale characteristics of the mixing layer. These interactions result in sensitivity to the initial perturbation spectrum, including the relative phases of the various modes, that persists long into the nonlinear phase of instability evolution.

I. INTRODUCTION

The question of the dependence of Richtmyer-Meshkov (RM) and Rayleigh-Taylor (RT) growth on the initial modal spectrum is at the heart of both astrophysical and ICF applications of compressible mix. This is particularly true for the deep nonlinear and transitional regimes, where linear and weakly nonlinear theories have become inapplicable but the similarity-based scaling arguments commonly applied to the turbulent regime are not yet necessarily valid. The deep nonlinear and transitional regimes must therefore bridge the gap between the earlier phases, where initial conditions have a strong and direct influence on the perturbation growth, and the turbulent regime characterized by self-similar growth independent of the initial spectrum.

In this paper, we present a mechanism whereby the unstable evolution of a strongly driven perturbed interface can depend critically on details of the initial mode spectrum. Specifically, we consider how the evolution of a long-wavelength mode is affected by a single short-wavelength component and the dependence of this effect on the relative phase between the modes.

These two-mode interfaces are driven in the regime of current experiments on the OMEGA laser.[1] The experiments, described in detail elsewhere,[2] use a 5 kJ 1 ns laser pulse to drive a Mach 15 blast wave into one end of a cylindrical target. The target consists of a heavier plastic pusher/ablator section and a lighter foam payload in contact along a perturbed interface. Aside from the initial perturbation, the earlier experiments differ from those discussed in this work only in the foam density – now 50 rather than 100 mg/cc (pre-shock density ratio now 0.035).

As the shock front crosses the interface at 1 ns and impulsively accelerates it up to about 70 km/s, it deposits vorticity, which drives RM growth. The interface then begins to decelerate, and does so for the 40 ns remainder of the experiment. The post-shock Atwood number remains nearly constant at $A^* = 0.7$. During the deceleration phase, the interface is RT unstable. Shock-deposited vorticity (RM) dominates the

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perturbation growth for the first couple of nanoseconds, while acceleration-induced growth (RT) dominates at later times.

II. SIMULATIONS

The 2D radiation-hydrodynamics code CALE.[3] is used for the simulations. The relative importance of several numerical and physical factors for achieving good agreement between simulation and experiment were considered in detail previously,[4] and the simulations in this work are run in accordance with those results. We note here only that the simulations are run in Eulerian mode, reflecting boundary conditions (BC's) are specified in the transverse direction, and the transverse resolution is 120 points per wavelength (ppw) of the lowest-l mode.

We now consider the same single-mode perturbation studied previously (50 μm wavelength and 2.5 μm initial amplitude), and study the effect of a single short wavelength component on its evolution. The scale of the secondary mode (mode 10) is one-tenth that of the primary mode (mode 1), or 5 μm in wavelength and 0.25 μm in initial amplitude. The relative phase of mode 10 with respect to mode 1 is either 0 or $\pi/2$. The symmetry of the single mode and in-phase cases allows us to limit the computational domain to one half of the mode 1 wavelength. In the out-of-phase case, we include four mode 1 wavelengths. We then technically have modes 4 and 40, but because the ratio of their wavelengths is an integer, no modes lower than 4 can be generated via mode coupling.

III. MODEL PREDICTIONS

Because of the strong drive and somewhat large initial amplitudes (pre-shock $a/\lambda = 0.05$ and post-shock $a^*/\lambda \approx 0.02$), the linear approximation is quickly invalidated. Mode 1 becomes nonlinear within 1 ns (of a 40 ns experiment) and mode 10 is nonlinear virtually instantaneously. In the two-mode cases, the bubble merger process proceeds rapidly. The ten small-scale bubbles per mode 1 wavelength present at 2 ns merge into one by 10 ns.In the similarly short early nonlinear phase, mode coupling is present but weak, and Haan's spectral model[5] is valid. Harmonic generation (of modes 2 and 20) introduces spike-bubble asymmetry, with spikes growing faster than bubbles. Modes 1 and 10 couple to generate modes 9 and 11, which allow for 3rd order coupling back to mode 1. Ofer *et al*[6] found that once a mode has reached its saturation amplitude, it no longer contributes to the growth of longer wavelength modes. However, they also found that a short wavelength mode can act as an affective density gradient at the interface to somewhat stabilize the primary mode.[7] According to their analysis, the mode 1 linear RT growth rate is reduced by 10% when $(a/\lambda)_{10}$ reaches about 0.3, or about the time mode 10 reaches its saturation velocity.

In order to make some prediction of the late-time dependence on the initial phases, we have applied the modal model of Ofer *et al*.[6] Our treatment of phases is more general than in the original implementation, which effectively allowed only for cosine modes with phases of only 0 and π. The resulting model predicts a negligible effect of mode 10 on the evolution of mode 1 regardless of the phases. Therefore the main effect of mode 10 on the evolution of mode 1 should be a reduction of its growth due the effective density gradient provided by mode 10.

IV. SIMULATION RESULTS

The averaged amplitude histories for the different phase realizations are shown in Fig. 1 along with the mode 1 prediction of Oron *et al*'s buoyancy-drag model.[8] In order to facilitate comparison with the model, the effect of target decompression has been removed. As expected, the linear phase lasts no more than about 1 ns. Mode 10 remains sufficiently small while the acceleration is large that the introduction of its amplitude into the buoyancy-drag model as a stabilizing density gradient results in virtually no change in the predicted mode 1 growth.

Additional detail can be obtained by considering the amplitude histories together with a plot of the spike-bubble amplitude asymmetry (see Fig. 2). Without the short wavelength component, the mode 1 amplitudes are as expected. The amplitudes begin to saturate as the drive decays away, with the spike amplitude significantly larger than that of the bubble. The two-mode-in-phase case begins similarly, but the bubble growth rate suddenly increases at about 11 ns. Late in time, the spike amplitude is somewhat reduced relative to the single-mode case (by less than 2%), while the bubble shows tremendous growth enhancement (about 65%). In fact the bubble amplitude is greater than the spike amplitude until about 30 ns. The out-of-phase case strongly differs from both the in-phase and the single-mode cases. The bubble growth is only slightly enhanced (by about 1%), but there is tremendous reduction of the spike growth (by about 60%). In both of the two-mode cases, the spike and bubble growth is nearly symmetric.

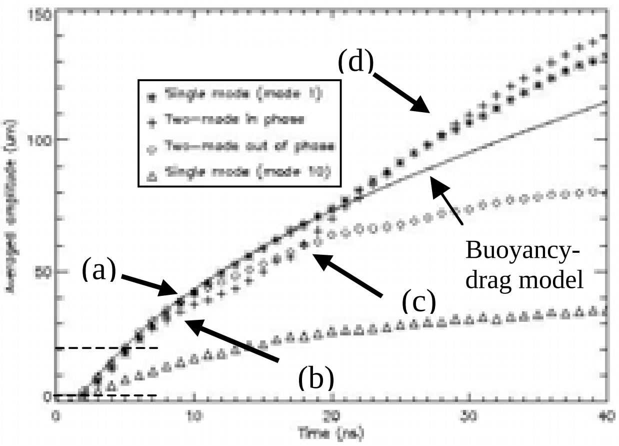

Figure 1: Spike-bubble averaged amplitudes corrected for decompression. (a) The bubble-merger-driven inverse cascade is complete by about 10 ns. (b) Early on, the growth is not strongly affected by short-λ "noise". (c) For the two-mode cases, sudden changes in growth rate occur at intermediate times. (d) After 20 ns, the phase-correlated (decorrelated) noise leads to growth enhancement (suppression) relative to single mode. The dashed lines show the single-mode saturation values ($a/\lambda = 0.4$) for modes 1 (upper) and 10 (lower).

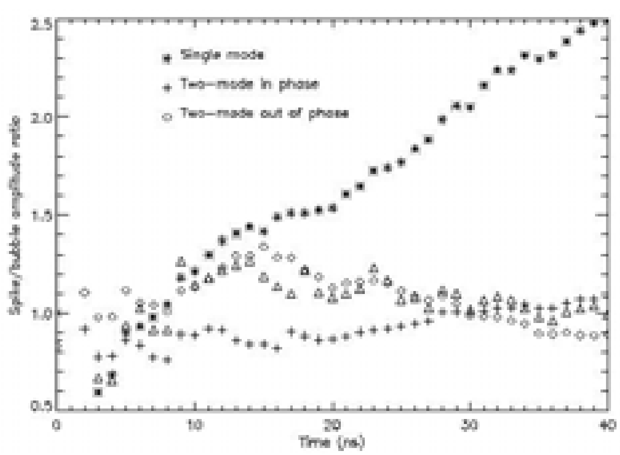

Figure 2: Ratio of spike to bubble amplitudes corrected for decompression. The single mode (mode 1) spike amplitude is significantly larger than that of the bubble at late times. Both two-mode calculations show nearly symmetric spike-bubble growth. The amplitudes are also nearly symmetric in the mode 10 single mode case (triangles) because of the Atwood number reduction due to the density gradient at the interface.

Thus mode 10 has little effect on mode 1 during the linear and early nonlinear phases of the instability evolution, but has a strong effect during the deep nonlinear phase when the driving acceleration has decayed to below 25% of its peak value. This does not appear to result from any affective density gradient provided by mode 10 at the mode 1 interface.

The operative mechanism can be understood by observing the interface as it evolves (see Fig. 3). At 2 ns (1 ns after shock refraction – see Fig 3a), the effect of mode 10 on mode 1 is clearly small. Mode 1 is just entering the early nonlinear phase ($a/\lambda = 0.1$) while mode 10 has already attained $a/\lambda = 0.4$. By 5 ns the shape of the primary spikes has been significantly altered by the presence of mode 10. The remaining secondary spikes (bubble merger is already underway) near the tips of the primary spikes have acquired a transverse velocity that is particularly pronounced in the in-phase case. In the in-phase case, transversely growing secondary spikes collide with one another at about 8 ns (Fig. 3b), driving premature bubble merger and producing upstream and downstream-directed jets. Since the collision direction is nearly perpendicular to the main flow direction, most of the collision energy is directed downstream. In the out-of-phase case, only a grazing collision occurs because alternating secondary spikes (at the primary spike tips) still have a significant upstream velocity component. As a result, half of these secondary spikes are directed downward and eventually strike the primary spike stalks at about 11 ns (Fig. 3c). This causes a sudden reduction in the spike amplitude growth rate and leads to the large reduction in spike growth relative to the single-mode case observed at late times. At about the same time, the downstream-directed jets produced in the in-phase case strike the inner surface of the primary bubble tips, thereby depositing energy that suddenly accelerates the bubble growth. At later times, KH activity near the primary spike tips effectively regenerates the smaller scales lost due to bubble merger. The process of secondary spike collision and jet formation can then continue, particularly in the in-phase case. Each new jet sends more spike material downstream into the primary bubble region, so that the coupling between KH and secondary spike interaction results in greatly enhanced mixing in the layer.

Between 26 and 30 ns, a large-scale vortex begins to form in the out-of-phase case. This

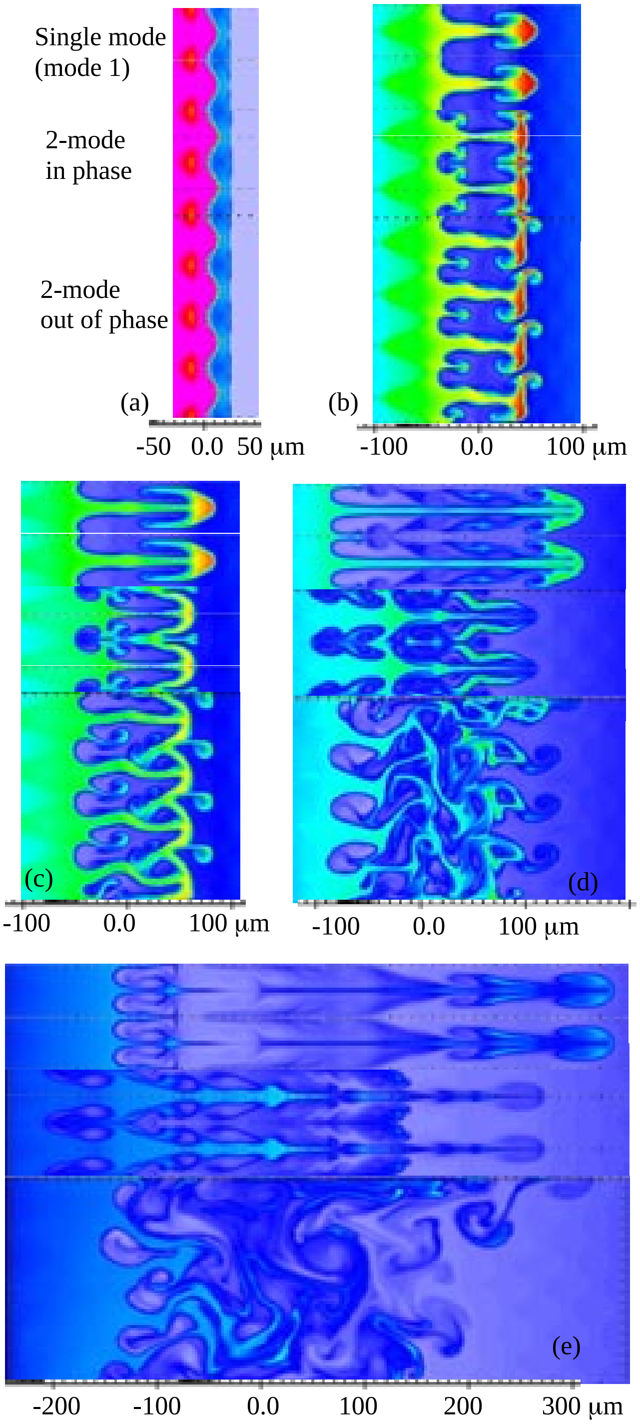

Figure 3: Density plots at (a) 2 ns, (b) 8 ns, (c) 11 ns, (d) 20 ns, and (e) 38 ns. Deflection of spikes can result in fast colliding spikes that drive premature bubble-merger and produce upwards and downwards-directed jets. Downwards-directed jets strike the inner surface of bubble tips, thereby depositing energy that accelerates bubble growth.

signifies that the edges of the computational domain have begun to influence the interface evolution all along the transverse direction. By this time, however, the spike amplitude in the out-of-phase case is already greatly reduced relative to the other cases and has nearly saturated.

At late times, there are large differences in the interface structure of all three cases (see Fig. 3e). Thus the large-scale features present during the late nonlinear instability evolution are strongly affected by the details of the initial conditions. Not only the presence of the short-wavelength mode, but also its phase, has a dramatic impact on the final state.

V. DISCUSSION

We hypothesize that a short wavelength secondary component can significantly affect the evolution of a longer wavelength primary mode when the aerodynamic drag pressure acting on the developing spikes is great enough that it not only determines the spike's terminal velocity, but also affects its shape. Considering the instability Mach numbers present in these simulations and experiments (M_{foam} = 0.15 - 0.20 and $M_{plastic}$ = 0.25 - 0.40), we can only place an upper bound of about M > 0.1 on when these effects might appear. The drag pressure is related to the thermal (or interface) pressure P_i through the expression

$$\rho_s^* v_s^2 = \gamma_s M^2 P_i, \qquad (1)$$

where M is the instability Mach number, v_s is the spike velocity, and ρ_s and γ_s are the density and adiabatic index of the spike material. When the Mach number is equal to 0.3 ($M^2 \approx 0.1$), the drag pressure is about one-tenth of the thermal pressure. In our case, the local pressure increase in secondary bubble regions results in partially redirecting the growth of secondary spikes into the transverse direction. This is reminiscent of the pinching effect noted by Li to occur during bubble merger, which can result in bubble-growth suppression due to an affective reduction in Atwood number.[9] But in our in-phase case, the head-on collision of two fast-moving secondary spikes produces an even faster downstream-directed jet. When it strikes the inner surface of the primary bubble, its ram pressure in the bubble frame is 0.45 Mbar – roughly equal to the 0.50 Mbar thermal pressure of the plastic at the bubble position. It is therefore able to penetrate a significant distance into the plastic, thereby enhancing the bubble growth. Although there is no head-on collision of secondary spikes in the out-of-phase case, the redirected spikes are still sufficiently energetic to essentially punch through the primary spike stalks.

Whether or not redirected secondary spikes collide with each other or with primary spike stalks depends on the degree of phase coherence. Interfaces consisting of periodic arrays of spikes are more likely to evolve into colliding spikes. Since real systems are very unlikely to exhibit high degrees of phase coherence, the perfect symmetry enforced by the in-phase calculation is arguably rather unphysical. There are, however, important implications for simulations. In multimode RT and RM calculations, the domain is often limited to a subsection or wedge of the full system with reflecting boundary conditions. In order to avoid unphysical effects at the boundaries, the initial perturbation spectrum sometimes includes only modes whose wavelengths are integer fractions of the full domain. The “random phase” assignment then amounts simply to a random assignment of plus or minus one to the amplitude of each mode. Since such spectra are actually characterized by a high degree of phase coherence, these simulations, if strongly driven, could significantly over-predict the growth of the mixing layer.

The interaction of redirected spikes represents a coupling between transverse and parallel motions and a complicated nonlinear transfer of energy from spikes to other spikes (driving premature bubble merger) and to bubbles (contributing to increased spike-bubble symmetry). Coupling of this process with the KH instability results in additional coupling between and generation of scales and greatly enhanced mixing in the layer. Since turbulence requires the development of a broad inertial range of scales, this will likely decrease the time to transition.

The result that large-scale features present during the late nonlinear instability evolution are strongly affected by details of the initial conditions must be reconciled with the expectation that, at some point near or after transition, the mixing layer will begin to grow at a rate that is independent of the initial spectrum. If this is correct, then the memory of the initial conditions must somehow be erased. The observed dependence would in that case be a transient phenomenon that would eventually disappear as the bubble size distribution settles into a scale invariant form. However, this argument requires the continual emergence of larger scales and depends on the existence of a sustained drive. In our case, the combination of a decaying drive and continuing decompression means that transients can effectively be “frozen in” to the flow and thereby persist to late times.

Finally, we note that we expect the jet effect to be significantly smaller, if not altogether absent, in 3D. Consequently, 3D calculations are planned to study the effect of initial conditions on determining the time to transition and the properties of the subsequent turbulent flow.

VI. ACKNOWLEDGEMENTS

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